**Problem 8**

Show that G(F) is non abelian for any n >= 2 and any F

Proof:

We note that every field has an additive identity 0 and a distinct multiplicative identity 1, so by restricting the proof to using only two values from F, we ensure the result will hold for any F.

Use induction on n. We already proved the base case for n = 2

Now assume that (F) is non-abelian for some n >= 3, and let A and B be the non-commuting members of (F). Now using the concept of block matrices, we have that

= =

Therefore is non abelian as we know that AB and this completes the proof.

**Problem 9**

Prove that the binary operation of matrix multiplication of 2X2 matrices with real number entries is associative.

Proof:

Consider the following sequence of operations:

] = = =

----(1)

And now consider the following series of operations:

) = = which is the same as (1)

Hence, we say that the binary operation of matrix multiplication is associative.

**Problem 10**

Let G =

1. Compute the product of to show that G is closed under matrix multiplication

Solution:

We have the following: = so G is closed under multiplication

1. Find the matrix inverse of and deduce that G is closed under inverses

Solution:

We use the explicit formula for a 2X2 matrix inverse. We note the determinant of the matrix is the product of the diagonal entries. So we have:

= and this is an upper triangular matrix

1. Deduce that G is a subgroup of ( R)

Solution:

A subgroup is a subset that is closed under multiplication and inverses (considering the group operation). By a) and b) G is a subgroup of ( R)

1. Prove that the set of elements of G whose diagonal entries are equal (a = c) is also a subgroup of (R)

Solution:

Call this set P , we have =

And =

H is closed under matrix multiplication and inversion, so H is a subgroup of (R)

**Problem 11**

Let H(F) =

X = { and Y = be the elements of H(F)

1. Compute the matrix XY and deduce that H(F) is closed under matrix multiplication. Exhibit explicit matrices such that XY

Solution:

We have that:

XY =( ) () =(

So, H(F) is closed under multiplication. Moreover, in order to tackle the non-abelian condition, we have that

So, H(F) is always non abelian

1. Find an explicit formula for the matrix inverse and deduce that H(F) is closed under inverses

**Solution**:

Let Z =

By performing the multiplication, XZ = ZX = I where I is the 3X3 identity matrix. It follows that Z = and since Z

c) Prove the associative law for H(F) and deduce that H(F) is a group of order

**Solution**:

=

= ()

= (, so multiplication in H(F) is associative. Thus H(F) is a group of order

Considering the matrix X above, if |F| = n < then each of a, b and c can take any of n values each. So |H(F)| =

d) Find the order of each element of the finite group H(Z/2Z)

Solution:

Obviously |I| = 1. For the rest, = = = I

So these have the order 2

= = I and these also have order 2

However, = = I and so the order is 4 in this case

|H(Z) /2Z| = = 8, so these are the elements of the group.

1. Prove that every nonidentity element of the group H( R ) has infinite order

**Solution**:

We will show by induction on n that [ = —(1)

Since any non-identity element has one of a, b or c nonzero, this will be enough to show that the element has infinite order.

The base case n = 1 is evident.

Assume the truth for some n >= 1

Then [ = (

=

So (1) holds for all positive n and hence the result follows.